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THE JET PROPULSION LABORATORY
METHOD OF TRACKING
LUNAR PROBES

Russell E. Carr

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JET PROPULSION LABORATORY
~~A~~ Research Facility of
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THE JET PROPULSION LABORATORY METHOD OF TRACKING LUNAR PROBES*

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I. INTRODUCTION

In this description of lunar-probe tracking, a small conical object is visualized speeding through space and spinning on its axis. The instruments aboard the object are collecting valuable scientific information and telemetering the information into space. An observation station on Earth must know in what direction its narrow beam antenna must be pointed in order to receive the signals being transmitted. Unless this beam is properly pointed at the proper time, the information will be lost. This directional aspect of tracking must take place in real time.

Then, a vast amount of data which has been collected by successfully pointing an antenna throughout the lifetime of the power supply carried by the probe is pictured. The time at which each data point was transmitted is known, but the place from which it came is yet to be determined. It is evident that the direction from which the signal came is not sufficient; the actual path of the probe through space is required. It may even be necessary to correlate the data with the

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spacial attitude of the missile axis at the time of transmission. These aspects of tracking, however, may be completed at a more leisurely rate.

In any event, a successful tracking program must provide for both the reception of the signals and the determination of the position and attitude of the probe as a function of time.

II. METHOD OF TRACKING

Radio telescopes to be used in tracking probes are commonly mounted in one of two manners. In either instance, two degrees of freedom are available. One axis is capable of allowing a 360-deg rotation, whereas the second axis (making a right angle with the first axis) allows rotations of plus or minus 90 deg. In what is termed an elevation-azimuth mounting, the axis of the 360-deg rotation is vertical. In the type of mounting called equatorial, this axis is parallel to the Earth's polar axis. In elevation-azimuth mounting, the local bubble-vertical is used more frequently than the local radial vertical from the center of the Earth.

When elevation-azimuth mounting is used, the angle of rotation about the local vertical is called the azimuth angle, σ. The other angle of rotation corresponds to the angle measured from the local horizontal and is called the elevation angle, γ (see Fig. 1).

When equatorial mounting is the method used, the angle of rotation about the axis parallel to the Earth's polar axis is called the local right ascension, α. The other angle of rotation corresponds to the angle measured from the local equatorial plane and is called the local declination, δ (see Fig. 2).

If both the position of the probe and the location of the radio telescope are known, the values of α, δ, γ, and σ may be determined. As will be discussed subsequently, however, the apparent values of these angles will not be their geometric values. In order to obtain the apparent values, the geometric values must be corrected for refraction and various station anomalies.

In order to facilitate operation of the gear device of the telescope, it is considered desirable to calculate the time rate also of α, δ, γ, and σ.

In order to successfully handle the information-receiving aspect of tracking, all radio telescopes must be provided with the appropriately corrected values of (α, δ) or (γ, σ) angles. Once the signal is acquired by a particular station, an automatic locked loop may be available to provide for automatic directional tracking until the probe goes below the station's horizon or beyond the range of the antenna.

When the powered portion of the flight has terminated, the path of the probe in space is defined by six parameters. Defining some particular time after powered flight as injection time t_0, the Earth fixed position and velocity components are used for the six parameters:

R_0 distance from center of the Earth to probe

φ_0 geocentric latitude of the probe

θ_0 longitude of the probe

v_0 magnitude of Earth-fixed velocity of probe

γ_0 elevation angle of Earth-fixed velocity (geocentric horizontal)

σ_0 azimuth angle of Earth-fixed velocity

These injection conditions serve as initial conditions to use with the equations of motion of the probe, and subsequent position and velocity components at time t_0 are obtained by numerical integration of the equations of motion. The actual numerical integration is carried out in space-fixed rectangular coordinates, and appropriate transformations are used to provide for other types of input and output.

The equations of motion include accelerations resulting from the Earth's oblateness as well as accelerations accounting for the gravitational effects of the Sun and the Moon.

In order to determine the path best fitting the observations, a sequence of estimates of the injection conditions is used.

The first estimate of the injection conditions is arrived at in one of two manners: either the nominal values of the injection conditions from a pre-flight standard are used, or the booster-cutoff conditions obtained from down-range instrumentation are used as initial conditions for a numerical integration through a nominal high-speed cluster.

The subsequent estimates are based on an iterative procedure. Using

$$R_0^{(k)},\ \varphi_0^{(k)},\ \theta_0^{(k)},\ v_0^{(k)},\ \gamma_0^{(k)},\ \sigma_0^{(k)}$$

to indicate the kth estimate of the injection parameters, the (k + 1)st estimate

$$R_0^{(k+1)},\ \varphi_0^{(k+1)},\ \ldots,\ \sigma_0^{(k+1)}$$

is obtained by adding on a correction obtained from correlating the predicted values of the observables at N successive times

$$t_1,\ t_2,\ .\ .\ .\ ,\ t_N$$

with the actual values observed at those times.

The types of observables assumed possible at the ith observation station are the (α_i, δ_i) angles of an elevation-azimuth radio telescope, the (γ_i, σ_i) angles of an equatorial-mount radio telescope, and a frequency count, f_i, from a doppler station.

In actual fact, three stations exist:

Cape Canaveral which has a doppler receiver only.

Puerto Rico, which has a doppler receiver and a small 10-ft dish elevation-azimuth mount radio telescope.

Goldstone Lake, located at Camp Irwin, California, which has a doppler receiver and a large 85-ft dish equatorial-mount radio telescope.

The primary function of the relatively small-range Puerto Rico antenna is provision of data to predict the initial Goldstone acquisition of the signal. Subsequent Goldstone acquisitions should be predictable from the previous ones.

In the instance of PIONEER IV, the range of the Goldstone antenna allowed for the reception of signals (when properly pointed) at distances greater than that of the Moon.

Unfortunately, drift of the transmitter in the probe made the doppler data unfit for angular predictions. Subsequent discussion of corrections made on predicted values is therefore confined to angular-type data.

Disregarding any observational errors, the apparent values observed for α, δ, γ, and σ would still not agree with the geometric values. With any calculated values for these quantities, then, corrections are applied which account for refraction and certain station anomalies.

The correction for refraction is not a simple procedure. The index of refraction, n, is not too well known and is dependent upon such factors as the temperature, the humidity, and the frequency of the signal being transmitted. At any given time, n can be thought of as a function of position. The actual path of a signal from the probe to the observation station is given theoretically as that path C such that the variation of the integral of nds over C is zero; that is, $\delta \int_C n ds = 0$.

No straightforward process is available for finding the apparent value of γ_i, which we call γ_i^*, even when the simplifying assumption is made that n is only a function of the height h above a sphere, concentric with the Earth, whose radius equals the distance from center of Earth to the observation station, and when the function n(h) is assumed known. A trial and error process of guessing at γ_i^* must be used; the correct value of γ_i^* is that value which yields the correct solution in an ordinary differential equation with two point boundary conditions. With such an assumption,

$$\gamma_i^* = f(\gamma_i, R-R_i)$$

Thus, if n(h) is assumed in advance, a table of the values of the function may be computed. When n = n(h), it may be concluded that

$$\sigma_i^* = \sigma_i$$

When γ_i^* and δ_i^* have been determined, a straightforward geometrical transformation yields the angles

$$\alpha_i^*, \quad \delta_i^* .$$

The elevation-azimuth radio telescopes are commonly aligned with the bubble vertical. Thus, another geometrical transformation leads to the values of

$$\gamma_i^{**}, \delta_i^{**}$$

The alignment of axes of the equatorial-mount telescopes is not dependent on the vertical, so

$$\alpha_i^{**} = \alpha_i^*$$

$$\delta_i^{**} = \delta_i^*$$

With both types of mounts, the antenna axes and the geometrical axes are slightly misaligned; therefore, the angles also have small additive bore-sight errors. Thus, we arrive at

$$\alpha_i^{***}, \delta_i^{***}, \gamma_i^{***}, \sigma_i^{***}$$

Although discussion of further corrections relating to the so-called polarization angle is of interest, it is not included in this presentation.

The theoretical basis for determining the corrections to the injection conditions has now been approached. To simplify notation, the predicted values of any one of the triple-starred quantities at time t_j, based on the kth estimate of the injection conditions, are designated, by $F_j^{(i,k)}$.

The true value is designated by $F_j^{(i)}$ and the observed value by $\hat{F}_j^{(i)}$.

The error of observation at time t_j is then ϵ_j, the difference between the observed value $\hat{F}_j^{(i)}$ and the true value $F_j^{(i)}$. Unfortunately, the true value of $F_j^{(i)}$ is not known. However, on the tacit assumption that the exact path of the probe would agree with the path calculated if the injection conditions were known, then, writing

$$F_j^{(i)} \equiv g\ (R_0,\ \varphi_0,\ \theta_0,\ v_0,\ \gamma_0,\ \sigma_0;\ t_j)$$

the estimate of the corresponding quantity at time, t_j, based on the kth estimate of the injection conditions, is given by

$$F_j^{(i,k)} \equiv g\ (R_0^{(k)},\ \varphi_0^{(k)},\ \theta_0^{(k)},\ v_0^{(k)},\ \gamma_0^{(k)},\ \sigma_0^{(k)};\ t_j)$$

Clearly, then, the true value, $F_j(i)$, can be expanded as a Taylor expansion in the six unknowns

$$R_0,\ \varphi_0,\ \theta_0,\ v_0,\ \gamma_0,\ \sigma_0,$$

about the point

$$R_0^{(k)},\ \varphi_0^{(k)},\ \theta_0^{(k)},\ v_0^{(k)},\ \gamma_0^{(k)},\ \sigma_0^{(k)}$$

Hence, the error of observation at any time t_j may be written as

$$\epsilon_j = \left[\hat{F}_j(i) - F_j(i,k)\right] - \frac{\partial F_j(i,k)}{\partial R_0^{(k)}}\left[R_0 - R_0^{(k)}\right] - \cdots - \frac{\partial F_j(i,k)}{\partial \sigma_0^{(k)}}$$

$$\times \left[\sigma_0 - \sigma_0^{(k)}\right] - \frac{1}{2}\frac{\partial^2 F_j(i,k)}{\partial R_0^{(k)2}}\left[R_0 - R_0^{(k)}\right]^2 - \cdots$$

It should be noted that the first bracketed term on the right of the equation is the difference between a value which is actually observed and a value which can be calculated with a high-speed computer. The other bracketed terms on the right are differences which, if they could be solved for, would give the exact increments to be added to the kth estimate of the injection conditions in order to obtain the true injection conditions. The right-hand side of the equation contains an infinite number of bracketed terms, but in the subsequent application, only the linear terms are eventually used. The coefficients of the linear terms are first partial derivatives at time t_j evaluated at

$$R_0^{(k)},\ \varphi_0^{(k)},\ \theta_0^{(k)},\ v_0^{(k)},\ \gamma_0^{(k)},\ \sigma_0^{(k)},$$

which means they can be calculated. The procedure used to date has been to approximate the partials by using one-sided differences. From the point of view of using a digital computer, this means choosing suitable increments

$$\Delta R_0,\ \Delta\varphi_0,\ \Delta\theta_0,\ \Delta v_0,\ \Delta\gamma_0,\ \Delta\sigma_0$$

and running seven trajectories simultaneously, one with initial conditions,

$$R_0^{(k)},\ \varphi_0^{(k)},\ \theta_0^{(k)},\ v_0^{(k)},\ \gamma_0^{(k)},\ \sigma_0^{(k)}$$

another with initial conditions,

$$R_0^{(k)},\ +\ \Delta R_0,\ \varphi_0^{(k)},\ \theta_0^{(k)},\ v_0^{(k)},\ \gamma_0^{(k)},\ \sigma_0^{(k)}$$

still another with initial conditions,

$$R_0^{(k)},\ \varphi_0^{(k)},\ +\ \Delta\varphi_0,\ \theta_0^{(k)},\ v_0^{(k)},\ \gamma_0^{(k)},\ \sigma_0^{(k)}$$

and so forth.

At all times, t_j, corresponding to actual observation from the ith station, the partial derivatives are then approximated, for example, by

$$\frac{\partial F_j^{(i,k)}}{\partial R_0^{(k)}} \simeq \frac{g\ (R_0^{(k)} + \Delta R_0,\ \varphi_0^{(k)},\ \ldots\ ,\ \sigma_0^{(k)};\ t_j) - F_j^{(i,k)}}{\Delta R_0}$$

and so on.

The assumptions on which the tracking is based are as follows: For lack of any better knowledge, it is assumed that the errors of observation, ϵ_j, are independent, and that they have a normal distribution about a zero mean with a standard deviation σ_j.

It is further assumed that the ratio of the σ_j for any two times is known; therefore $\sigma_j = \sigma/w_j$, where w_j is known. It is evident that, at any time no information is available concerning w_j, all values of w_j can, for the time being, be set equal to one. It is also clear that when all the data are available and a more leisurely approach is possible, the data can be examined to obtain an estimate of w_j.

It is assumed that the best fit to the data is that which maximizes the likelihood function of the observational errors. In the instance of one observable, the last-mentioned assumption means that if the ϵ_j is multiplied by w_j, the square of the product $w_j \epsilon_j$ is summed over all j, and the derivatives of this sum with respect to each of the differences

$$\left[R_0 - R_0^{(k)}\right], \left[\varphi_0 - \varphi_0^{(k)}\right], \cdots, \left[\sigma_0 - \sigma_0^{(k)}\right],$$

is set to zero, then a system of six non-linear equations in these differences is obtained. As each of these six equations involves an infinite series in the above differences, the set cannot be solved explicitly.

At this time then, the linearization hypothesis is made: It is assumed that, if all but the linear terms in the differences are dropped and the differences replaced by

$$\delta R_0^{(k)}, \delta\varphi_0^{(k)}, \cdots, \delta\sigma_0^{(k)}$$

respectively, then one can solve the linear system, get the estimates

$$R_0^{(k+1)} = R_0^{(k)} + \delta R_0^{(k)}, \cdots, \sigma_0^{(k+1)} = \sigma_0^{(k)} + \delta\sigma_0^{(k)}$$

use the new estimates and iterate until convergence. It is further assumed that the values to which convergence is made will be the maximum likelihood estimate of

$$R_0, \varphi_0, \theta_0, v_0, \gamma_0, \sigma_0$$

Theoretically, it would appear that one type of data with sufficient observations would enable the determination of the initial conditions. In any event, it must be assumed that measurements of sufficient number and quality are available for determining the six initial conditions. From a practical viewpoint, it is clear that any time angular data are available from a radio telescope, two types of data are available. Thus, if only one angle is used, valuable information is being discarded.

When combining data of two or more types, the maximum likelihood approach leads to the assumption that each data type will be weighted inversely as its standard deviation σ_j at time t_j. That is, each data type will have its own σ, and, in addition, at each time t_j the particular observable will also have its own weight w_j.

The effort to obtain corrections to the injection estimates using, first, preliminary estimates based on each data type separately, then combining the preliminary estimates, has not proved nearly so satisfactory as obtaining one estimate from combined data. Fifteen minutes of early angular data from Puerto Rico at 10-sec intervals have proved extremely useful in pinning down the initial conditions for pointing, when treated as combined data. When treated separately, the same data did not produce very satisfactory results.

III. SUMMARY

It may be of interest to summarize the current status of the application of the tracking program to the two lunar-probe efforts designated as PIONEER III and PIONEER IV.

In the first case, the over-all result was approximately a 38-hour flight. All of the telemetered data were successfully received. The Puerto Rico station received the signals from the probe within a few minutes of launching. Eight hours of data from Puerto Rico were received; the signal at Goldstone was picked up one-half hour before it was lost at Puerto Rico, and the loss at Puerto Rico occurred when the distance exceeded the range of the Puerto Rico antenna. The Goldstone station, which has the more powerful antenna, received signals for another eight hours, losing contact when the probe went below the horizon. Fourteen hours later, when the probe came closer to the Earth, the Puerto Rico antenna was able to pick up the signals, but lost them again eight hours later when the probe went over the horizon as it plunged to Earth.

The procedure for using combined data was not available at the time of PIONEER III, and the tracking was more or less experimental. With much perseverance, the Goldstone acquisition and the Puerto Rico re-acquisition were successfully predicted.

As the procedure for using combined data was available for PIONEER IV, the pointing of the antenna was a smooth operation, Goldstone acquisition and re-acquisition were successful, and all telemetered data above the Goldstone horizon were received until the power supply at the probe gave out. The last data were received on the fifth day, at which time the probe had exceeded twice the distance to the Moon.

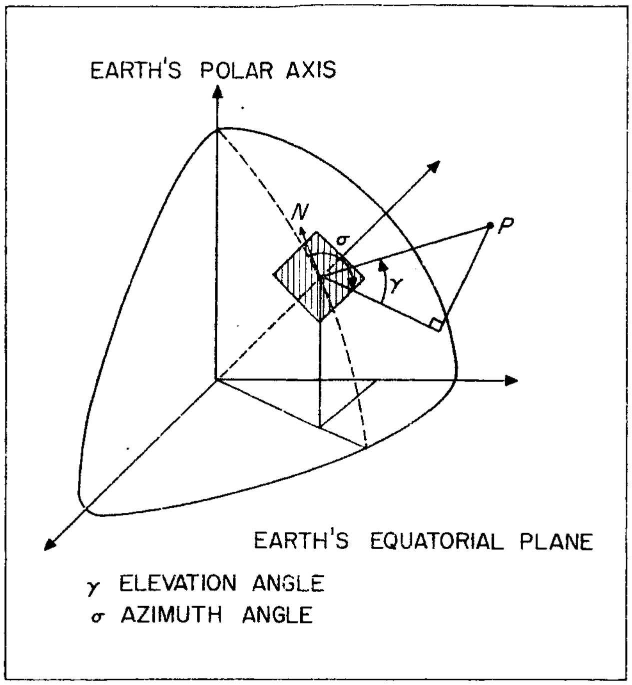

Fig. 1. Elevation-Azimuth Mounting

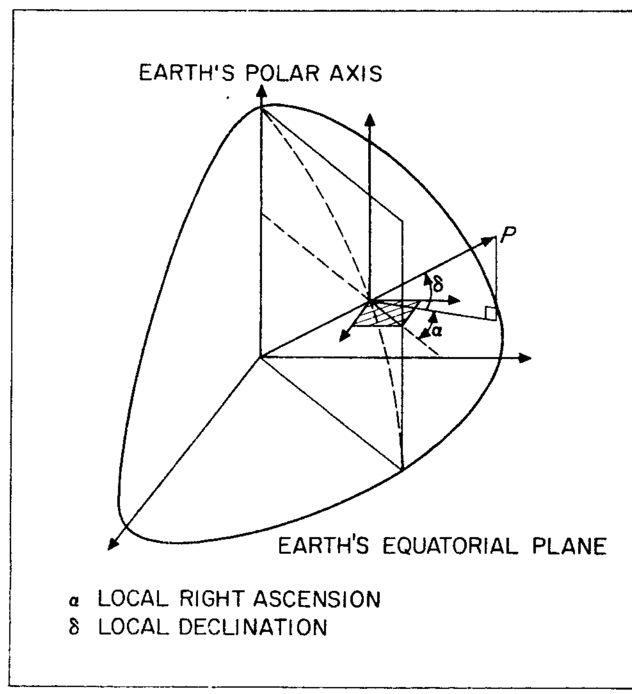

Fig. 2. Equatorial Mounting